

MATHEMATICS & STATISTICS-I

Time : 2 Hrs.

OCTOBER 2009

Marks : 40

- Note :** (i) All questions are compulsory.
 (ii) Figures to the right indicate full marks.
 (iii) Answers to every question must be written on a new page.

Q. 1. (A) Attempt any ONE of the following :

- (i) Differentiate w. r. t. $x : x \cdot \log (\sin x)$ (2)
- (ii) Find $\frac{dy}{dx}$, if $x = a \cos \theta$, $y = b \sin \theta$ (2)

(B) Attempt any ONE of the following :

- (i) State whether the following sentences are statements in logic : (2)
 If so write the truth value.
- (a) Two parallel lines intersect each other.
 (b) 3 is root of the equation $x^2 - 5x + 6 = 0$
- (ii) Express the truth of each of the following statements by Venn-diagram : (2)
- (a) No policemen are thieves.
 (b) Some students are hard workers.

(C) Attempt any ONE of the following : (4)

- (i) Evaluate : $\int \frac{2x^2 - 1}{(x^2 + 4)(x^2 + 5)} dx$ (ii) Evaluate : $\int \frac{x}{1 + \cos x} dx$

Q. 2. (A) Attempt any ONE of the following :

- (i) Write the following statements in symbolic form. (2)
- (a) Reshma is tall or she is short and beautiful.
 (b) Amitabh is tall but not handsome.
- (ii) Prove by using truth table : $\sim (p \vee q) \equiv \sim p \wedge \sim q$ (2)

(B) Attempt any ONE of the following :

- (i) Find x, y, z if $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} [1 \ 2] \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ (2)

- (ii) Find the inverse of matrix $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ (2)

(c) Attempt any ONE of the following :

- (i) Solve : $\frac{dy}{dx} = (9x + y + 2)^2$ by putting $9x + y + 2 = u$ (4)
- (ii) The rate of growth of bacteria is proportional to the number of bacteria present. If the original number N doubles in 3 hours, show that the number will be $4N$ in 6 hours. (4)

Q. 3. (A) Attempt any ONE of the following :

- (i) Evaluate : $\lim_{x \rightarrow 2} \left[\frac{1}{x-2} - \frac{2}{x(x^2 - 3x + 2)} \right]$ (2)

(ii) Find k , if the function

$$f(x) = \frac{x^3 - 64}{x - 4}, \text{ for } x \neq 4 \quad (2)$$

$$= k, \text{ for } x = 4$$

is continuous at $x = 4$

(B) Attempt any ONE of the following :

(i) From the differential equation by eliminating A and B from the relation

$$y = Ae^x + Be^{-x}$$

(ii) Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{x}{y} \text{ when } x = 3, y = 2$$

(C) Attempt any ONE of the following :

(i) Evaluate : $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x^2}$

(ii) If f is continuous at the point $x = 0$,

$$\text{where } f(x) = \frac{\sin 4x}{5x} + \alpha, \text{ for } x > 0$$

$$= x + 4 - \beta, \text{ for } x < 0$$

$$= 1, \text{ for } x = 0$$

then find α and β

Q. 4. (A) Attempt any ONE of the following :

(i) Write the negation of

(a) All men are animals. (b) All men take water and meal.

(ii) Test whether the statement $(p \wedge q) \vee \sim q$ is a Tautology, Contradiction or Contingency.

(B) Attempt any ONE of the following :

(i) Differentiate w.r.t. x : $x^2 y^2 = x^2 - y^2$

(ii) If $y = \cot^{-1} \left(\frac{1 + \cos x}{\sin x} \right)$, then find $\frac{dy}{dx}$

(C) Attempt any ONE of the following :

(i) Differentiate w.r.t. x : $x^x + x^{\sin x}$

(ii) Find the approximate value of $\tan^{-1}(0.999)$

Q. 5. (A) Attempt any ONE of the following :

(i) Express the following equations in matrix form and solve by Reduction method

$$x + 3y = 7 \quad 2x + y = 9$$

(ii) Show that $A^2 - 4A + 3I = 0$

$$\text{Where } A = \begin{bmatrix} 1 & 3 \\ 0 & 3 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(B) Attempt any ONE of the following :

(i) Evaluate : $\int e^x [1 + \tan x + \tan^2 x] dx$

(ii) Evaluate : $\int \frac{dx}{x^2 + 6x + 10}$

(C) Attempt any ONE of the following :

(i) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$

(ii) Evaluate : $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$

