

Mathematics & Statistics - I

Time : 2 Hrs.

October - 2008

Marks : 40

Q. 1. (A) Attempt any one of the following :

(i) Write the following statement in the symbolic form and state its truth value :

It is false, that if a quadrilateral is a rhombus then it is a parallelogram.

(ii) Write the negations of the following statements without using the phrase

'It is false that' or 'It is not true that' :

(a) Some teachers are sincere.

(b) $2 \times 3 = 6$ and $2 + 3 \neq 4$.

(B) Attempt any ONE of the following :

(i) Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left(\frac{2\sqrt{x}}{1-x} \right)$. (ii) Find $\frac{dy}{dx}$ at $t = 3$, if $x = at^2$ and $y = 2at$.

(C) Attempt any ONE of the following :

(i) Solve the differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$, using the substitution $x+y = v$.

(ii) Form the differential equation by eliminating the arbitrary constants A and B from :

$$y = Ae^{4x} + Be^{2x}$$

Q. 2. (A) Attempt any ONE of the following :

(i) Evaluate $\int xe^x dx$ (ii) Evaluate $\int \frac{1 - \tan x}{1 + \tan x} dx$

(B) Attempt any ONE of the following :

(i) Write the equations $x + y = 5$ and $2x - y = 4$ in the matrix form and solve them using the method of reduction.

(ii) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & -1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix}$, find whether $(AB)^{-1}$ exists.

(C) Attempt any ONE of the following :

(i) Evaluate $\int_0^4 \frac{1}{x + \sqrt{16-x^2}} dx$ (ii) Evaluate $\int_0^1 x^2 (1-x)^{\frac{3}{2}} dx$

Q. 3. (A) Attempt any ONE of the following :

(i) If 'p' is a true statement and 'q' and 'r' are false statements, find the truth values of

(a) $[p \wedge (\sim q)] \rightarrow r$ (b) $p \vee (q \vee r)$.

(ii) Find whether the following statement is a tautology or a contradiction or neither : $(p \wedge q) \leftrightarrow \sim(p \wedge q)$

(B) Attempt any ONE of the following :

(i) If $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 7 \\ 4 & 5 & 6 \end{bmatrix}$.

find the matrix $(3A - B')$, where B' is the transpose of B.

(ii) If $A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, compute the matrix $A^2 + 2A$.

(C) Attempt any ONE of the following :

(i) Evaluate : $\int \sqrt{\frac{x+1}{x+3}} dx$ (ii) Evaluate : $\int \frac{1}{\sin^2 x + 2\cos^2 x + 3} dx$

Q. 4. (A) Attempt any ONE of the following :

- (i) If the price per unit of a commodity is given by $p = 10 - \frac{x}{4000}$, where x is the number of units sold, find the marginal revenue when $x = 4000$
- (ii) The total cost function, C is given by $C = x^2 - 14x + 8$, where x is the output. Find the output for which the total cost is increasing.

(B) Attempt any ONE of the following :

- (i) Write the duals of the following statements :
- (a) The film receives an award for its story or for its direction.
- (b) $(\sim p \wedge q) \vee (\sim r)$
- (ii) State giving reason, which of the following statements have the same meaning :
- (a) If monsoon is good then farmers are happy.
- (b) If farmers are happy then monsoon is good.
- (c) If monsoon is not good then farmers are not happy.
- (d) If farmers are not happy then the monsoon is not good.

(C) Attempt any ONE of the following :

(i) Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x + a^{-x} - 2}{\sin 4x \log(1+x)} \right)$.

- (ii) Find k , if the function f defined as

$$f(x) = \frac{2 - \sqrt{3 + \cos kx}}{x^2}, x \neq 0.$$
$$= 2, x = 0$$

is continuous at $x = 0$

Q. 5. (A) Attempt any ONE of the following :

- (i) If the rate at which the profit of a company is increasing is inversely proportional to the profit at that time, obtain the expression for the company's profit at any time 't'. (Profit = 0 when $t = 0$)
- (ii) If the marginal cost of a commodity is equal to its average cost, prove that the total cost is directly proportional to the number of units produced.

(B) Attempt any ONE of the following :

- (i) Discuss the continuity of the function f defined as

$$f(x) = \left(1 + \frac{5x}{3} \right)^{\frac{1}{x}}, \text{ if } x \neq 0$$
$$= e^{\frac{5}{3}}, \text{ if } x = 0$$

(ii) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{(\pi - 2x)}$

(C) Attempt any ONE of the following :

- (i) If $e^x + e^y = e^{x+y}$ Prove that $\frac{dy}{dx} = -e^{y-x}$
- (ii) Find $\frac{dy}{dx}$, if $y = \frac{(1+x)^x}{\tan x}$.

