

# Mathematics & Statistics - I

Time : 2 Hrs.

March - 2009

Marks : 40

**Q.1 (A) Attempt any ONE of the following :**

(2)

(i) Solve the differential equation :

$$\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$$

(ii) Form the differential equation by eliminating arbitrary constants 'a' and 'b' from the relation

$$y^2 = ae^{2x} + be^{-2x}.$$

**(B) Attempt any ONE of the following :**

(2)

(i) State which of the following sentences is a statement :

(a) 14 is a perfect square.

(b) Give me a glass of water.

(ii) Construct the truth table for  $p \vee (p \wedge q)$

**(C) Attempt any ONE of the following :**

(4)

(i) Solve the differential equation  $x^2y dx - (x^3 + y^3)dy = 0$ . Put  $y = vx$ .

(ii) Find the particular solution of the differential equation :

$$(e^y + 1) \cos x dx + e^y \sin x dy = 0 \text{ given that } x = \frac{\pi}{4}, y = 0$$

**Q.2. (A) Attempt any ONE of the following :**

(2)

(i) Evaluate :  $\int x \log x dx$

(ii) Evaluate :  $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

**(B) Attempt any ONE of the following :**

(2)

(i) If truth value of p is 'T' and truth value of q is 'F', then find the truth value of

(a)  $p \rightarrow q$

(b)  $\sim p \wedge q$

(ii) Show that  $(p \wedge q) \rightarrow q$  is a tautology.

**(C) Attempt any ONE of the following :**

(4)

(i) Evaluate  $\int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$

(ii) Evaluate  $\int_2^3 \frac{x}{(x+2)(x+3)} dx$

**Q.3. (A) Attempt any ONE of the following :**

(2)

(i) If  $y = \frac{e^x - 1}{e^x + 1}$ , then find  $\frac{dy}{dx}$

(ii) If  $y = \sqrt{\cos(3x - 1)}$ , then find  $\frac{dy}{dx}$

**(B) Attempt any ONE of the following :**

(2)

(i) Let p : Sachin is a good boy.

q : Viraj is tall

State the Verbal form of the statements :

(a)  $p \wedge q$

(b)  $\sim p \rightarrow q$

(ii) Draw Venn diagrams to represent the truth of the following statements :

(a) No teacher is rich.

(b) All engineers are intelligent.

**(C) Attempt any ONE of the following :**

(4)

(i) Show that  $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + C$

(ii) Evaluate  $\int \frac{1}{5+4\cos x} dx$

**Q. 4. (A) Attempt any ONE of the following :**

(i) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1+x}{1-x} \right)^{\frac{1}{x}}$

(ii) Discuss the continuity of  $f(x)$  at  $x = 2$ , where

$$f(x) = \frac{x^2 - 4}{x - 2}, \text{ for } x \neq 2$$

$$= 4, \quad \text{for } x = 2$$

**(B) Attempt any ONE of the following :**

(i) If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ , show that  $A^2 - 5A - 21 = 0$

(ii) If  $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$  find  $|A B|$

**(C) Attempt any ONE of the following :**

(i) If  $y = \cot^{-1} \left( \frac{1 - 12x^2}{7x} \right)$ , then find  $\frac{dy}{dx}$

(ii) If  $x^3 \cdot y^2 = (x + y)^5$  then show that  $\frac{dy}{dx} = \frac{y}{x}$

**Q. 5. (A) Attempt any ONE of the following :**

(i) If  $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ , find  $A^2 - 7A$

(ii) Find the values of  $x$  and  $y$ , if

$$\begin{bmatrix} 1 & x & 0 \\ y & 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

**(B) Attempt any ONE of the following :**

(i) If  $x = a \cos \theta$ ,  $y = a \sin \theta$ , then find  $\frac{dy}{dx}$

(ii) If  $y = \sin x + \cos x$ , then show that  $\frac{d^2y}{dx^2} + y = 0$

**(C) Attempt any ONE of the following :**

(i) Evaluate:  $\lim_{x \rightarrow 1} \frac{\sqrt{3+x} - \sqrt{5-x}}{x^2 - 1}$

(ii) If  $f(x)$  is continuous at  $x = 0$ ,

$$\text{Where } f(x) = \frac{\sin 4x}{5x} + a, \quad \text{for } x > 0$$

$$= x + 4 - b, \quad \text{for } x < 0$$

$$= 1, \quad \text{for } x = 0$$

then find 'a' and 'b'.

