

Mathematics & Statistics - I

Time : 2 Hrs.

March - 2008

Marks : 40

Q. 1. (A) Attempt any ONE of the following :

- (i) Draw Venn diagrams to represent the following statements, assuming them to be 'true'.
(a) All doctors are honest. (b) Some doctors are honest.
- (ii) If $p \leftrightarrow q$ is a 'false' statement and $p \rightarrow q$ is a 'true' statement, write the truth value $[p \vee (\sim q)]$.

(B) Attempt any ONE of the following :

- (i) If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ prove that $|AB| = |A| |B|$
- (ii) Find the value of x and y , if

$$\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & 1 \\ 2 & 3 & 8 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(C) Attempt any ONE of the following :

- (i) The total cost 'C' of producing x items is given by $C = x^3 - 300x^2 + 12x$ find 'x' for which the marginal cost is decreasing.
- (ii) The total revenue 'R' and the total cost 'C' of a firm are given by $R = 380x - 3x^2$ and $C = 20x$ respectively where x is the quantity. If there is an error of 0.5% in measuring the quantity, find approximate the consequent error in the calculation of the profit, when the quantity is 10 units.

Q. 2. (A) Attempt any ONE of the following :

- (i) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$, find A^{-1}
- (ii) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, prove that $A^2 - 4A$ is a scalar matrix.

(B) Attempt any ONE of the following :

- (i) Write the negations of the following statements, without using the phrase 'It is false that' or 'It is not true that' :
(a) I like Mathematics or English. (b) If a quadrilateral is a rectangle then it is a parallelogram
- (ii) Write the dual and the negation of the statement $(p \vee q) \wedge r$.

(C) Attempt any ONE of the following :

- (i) Evaluate $\int \frac{dx}{5 - 4 \cos x}$
- (ii) Evaluate $\int \frac{x^2 + 14}{x^4 + x^2 - 2} dx$

Q. 3. (A) Attempt any ONE of the following :

- (i) Rewrite the statement : "The demand falls if and only if the price increases".
removing the indicative 'if and only if' and using the connectives 'not', 'and', 'or'.
- (ii) Find whether the statement $[\sim (p \vee q)] \vee p$ is a Tautology, Contradiction or neither.

(B) Attempt any ONE of the following :

- (i) Find $\frac{dy}{dx}$, if $y = x^{\sin x}$
- (ii) Find $\frac{dy}{dx}$, if $y = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$

(C) Attempt any ONE of the following :

- (i) Evaluate $\int_0^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx$ (ii) Evaluate $\int_0^1 \tan^{-1} x dx$

Q. 4 (A) Attempt any ONE of the following :

(i) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin^3 x}{\cos^2 x}$

(ii) Discuss the continuity of the function f defined as

$$f(x) = \frac{\sqrt{x+8} - 3}{x^3 - 1}, x \neq 1$$
$$= \frac{1}{3}, x = 1$$

at $x = 1$.

(B) Attempt any ONE of the following :

- (i) Find $\frac{dy}{dx}$ if, $x = a \cos \theta$, $y = a \sin \theta$. (ii) Find $\frac{dy}{dx}$ if, $y = (x^2 + 3x) \log x$.

(C) Attempt any ONE of the following :

(i) Solve the differential equation : $\frac{dy}{dx} = \frac{4x + 6y - 2}{2x + 3y + 3}$, by taking $2x + 3y = t$.

(ii) The rate of increase of the population of a city varies as the population at that time. In a period of 40 years the population increased from 4 lakhs to 6 lakhs. Show that in another 20 years the population will be 7.3482 lakhs.

$$\left[\text{Take } \sqrt{\frac{3}{2}} = 1.2247 \right]$$

Q. 5. (A) Attempt any ONE of the following :

- (i) Evaluate : $\int \frac{1}{1 + \cos x} dx$ (ii) Evaluate : $\int \frac{1}{x+x^3} dx$

(B) Attempt any ONE of the following :

- (i) Form the differential equation by eliminating the arbitrary constants a , b from $y = ae^{bx}$
- (ii) Verify that $y = A \sin 3x + B \cos 3x$ is the general solution of the differential equation $\frac{d^2y}{dx^2} + 9y = 0$

(C) Attempt any ONE of the following :

(i) Evaluate : $\lim_{x \rightarrow 0} \frac{e^{8x} - e^{5x} - e^{3x} + 1}{\cos 4x - \cos 10x}$

(ii) If f is continuous at $x = 0$, where

$$f(x) = x^2 + a \quad \text{for } x \geq 0$$
$$= 2\sqrt{x^2 + 1} + b \quad \text{for } x < 0$$

find a , b given that $f(1) = 2$

